First Law Calculations for Ideal Gas Expansion and Compression

relationships that apply to **ideal gasses** for all conditions with $w_{other}=0$
(some also apply **more generally**):

$\Delta U = q + w$ | $w = -\int P_{ext} dV$ | $PV = nRT$

$q_V = n\bar{C}_V \Delta T$ | $q_P = n\bar{C}_P \Delta T$ | $\bar{C}_P = \bar{C}_V + nR$

$H \equiv U + PV$ | $\Delta U_{any\ conditions} = n\bar{C}_V \Delta T$ | $\Delta H_{any\ conditions} = n\bar{C}_P \Delta T$

monatomic ideal gas: $\bar{C}_V = \frac{3}{2}R$ $\quad \bar{C}_P = \frac{5}{2}R$

reversible vs **irreversible** processes:

Reversible: $P_{ext}=P_{int}=P$ throughout expansion or compression

Irreversible: often P_{ext}=constant
(external pressure instantaneously change to P_{ext}
and gas expands/contracts from $P_{initial}$ to $P_{final}=P_{ext}$
doing work against P_{ext} =constant)

calculation of q, w, ΔU, ΔH for ideal gas:

1. any **isothermal** (ΔT=0, ΔU=0, ΔH=0, q=–w)
   a. *isothermal reversible*: $w=-\int P_{ext}dV=-\int_{V_1}^{V_2} \frac{nRT}{V}dV=-nRT\ln\frac{V_2}{V_1}$

   b. *isothermal irreversible* (against P_{ext} =constant):
   $$w=-\int PdV=-P_{ext}\int_{V_1}^{V_2} dV=-P_{ext}\left(V_2-V_1\right)$$

2. any **adiabatic** (q=0, w=ΔU)
   a. *adiabatic reversible*: (here one would be given $(T_1,P_1,V_1) \longrightarrow (P_2$ or $V_2)$:
   use relations among $P_1,T_1,V_1 \leftrightarrow P_2,T_2,V_2$ *to get final* T_2 :
   $$\frac{T_1^{\frac{\bar{C}_P}{R}}}{P_1}=\frac{T_2^{\frac{\bar{C}_P}{R}}}{P_2}$$
   $$P_1 V_1^{\frac{\bar{C}_p}{\bar{C}_V}} = P_2 V_2^{\frac{\bar{C}_p}{\bar{C}_V}}$$
   i
   $$T_1^{\frac{\bar{C}_V}{R}} V_1 = T_2^{\frac{\bar{C}_V}{R}} V_2$$

   for monatomic ideal gas: $\frac{\bar{C}_V}{R}=\frac{3}{2},\ \frac{\bar{C}_P}{R}=\frac{5}{2},\ \frac{\bar{C}_P}{\bar{C}_V}=\frac{5}{3}$

ii once T_2 and thus ΔT are known then:

$$\Delta U = n\bar{C}_V \Delta T,\ \ \Delta H = n\bar{C}_P \Delta T,\ w = \Delta U,\ q = 0$$

b. *adiabatic irreversible* (against P_{ext} =constant):

i here one would normally be given (T_1,P_1,V_1)⟶ (P_2=P_{ext}) and one would need to find T_2 to apply relationships for ΔU and ΔH

$$w = -\int_{V_1}^{V_2} P_{ext} dV = -P_{ext}\left(V_2 - V_1\right) = P_{ext}\left(\frac{nRT_1}{P_1} - \frac{nRT_2}{P_2}\right)$$

$$\Delta U = n\bar{C}_V\left(T_2 - T_1\right) = w$$

$$n\bar{C}_V\left(T_2 - T_1\right) = P_{ext}\left(\frac{nRT_1}{P_1} - \frac{nRT_2}{P_2}\right)$$

$$\textit{with } P_2 = P_{ext} \textit{ and solving for } T_2 \text{ (factoring out } n\text{)}$$

$$\left(\bar{C}_V + \frac{RP_{ext}}{P_{ext}}\right)T_2 = \left(\bar{C}_V + \frac{RP_{ext}}{P_1}\right)T_1$$

$$T_2 = \left(\frac{\bar{C}_V + \frac{RP_{ext}}{P_1}}{\bar{C}_V + R}\right)T_1$$

you should verify if this makes sense for P_{ext}>P_1 (compression) versus P_{ext}<P_1 (expansion)

ii once T_2 and thus ΔT are known then ;

$$\Delta U = n\bar{C}_V \Delta T,\ \ \Delta H = n\bar{C}_P \Delta T,\ w = \Delta U;\ q = 0$$